SUMMARY OF FACTORING TECHNIQUES

FOR ALL POLYNOMIALS

Factor out the Greatest Common Factor

Factor out the GCF
Divide each term by the GCF
Simplify

FOR TWO TERM POLYNOMIALS

Factoring the Difference of Two Squares

Make sure we can apply the formula: $a^2 - b^2 = (a+b)(a-b)$

Factoring the Sum or difference of Two Cubes

Make sure we can apply one of the following formulas:

Formula for factoring the sum of two cubes:
$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$.

Formula for factoring the difference of two cubes:
$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$.

Note- some people remember S.O.A.P. (Same Sign, Opposite Sign, Always Positive) to help them remember these formulae.

FOR THREE TERM POLYNOMIALS

Factoring Trinomials of the Form $ax^2 + bx + c$

Example: Factor $3T^2 + 8T - 35$ completely.

Factor out the GCF, if necessary. Not necessary in this case since the GCF is 1.

Identify $a = 3$, $b = 8$, and $c = -35$.

Multiply $a \cdot c$, in this case, $a \cdot c = 3 \cdot -35 = -105$.

Find all pairs of numbers that have a product of -105 and a sum of b. In this case, b=8. If no pairs satisfy this condition, our trinomial is prime, which means that it cannot be factored.

Pair	Product	Sum
$-1, 105$	$-1 \cdot 105 = -105$	$-1 + 105 = 104$
$1, -105$	$1 \cdot -105 = -105$	$1 + (-105) = -104$
$-3, 35$	$-3 \cdot 35 = -105$	$-3 + 35 = 32$
$3, -35$	$3 \cdot -35 = -105$	$3 + (-35) = -32$
$-5, 21$	$-5 \cdot 21 = -105$	$-5 + 21 = 17$
$5, -21$	$5 \cdot -21 = -105$	$5 + (-21) = -17$
$\mathbf{-7, 15}$	$\mathbf{-7 \cdot 15 = -105}$	$\mathbf{-7 + 15 = 8}$
$7, -15$	$7 \cdot -15 = -105$	$7 + (-15) = -8$

The pair that satisfies our condition is -7 and 15. So, we rewrite our original trinomial, $3T^2 + 8T - 35$, using that pair as follows:

$3T^2 + 8T - 35$	
$3T^2 - 7T + 15T - 35$	(Since $-7T + 15T = 8T$)
$T(3T - 7) + 5(3T - 7)$	(Factor by Grouping, see box to the right)
$(3T - 7)(T + 5)$	(Factor out the GCF of $3T - 7$, see box to the left)

FOR FOUR OR MORE TERMS (grouping)

EASY EXAMPLE

Rearrange the terms:
ax + b + a + bx = ax + a + bx + b.

Factor out an a in the first two terms, and a b in the second two terms:
ax + a + bx + b = a(x + 1) + b(x + 1)

Make sure what is in the two sets of parentheses is the same:
(x + 1) = (x + 1)? YES!

Now multiply what is in the parentheses by everything that is not in the parentheses:
a(x + 1) + b(x + 1) = (x + 1)(a + b)

HARDER EXAMPLE

In the problem, $a^2 + 2ab + b^2 - x^2$, recognize that
$a^2 + 2ab + b^2 = (a+b)^2$.

Rewrite $a^2 + 2ab + b^2 - x^2$ as
$(a+b)^2 - x^2$.

Now $(a+b)^2 - x^2$ is a difference of two squares, so
$(a+b)^2 - x^2 = (a+b+x)(a+b-x)$